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Modeling wind speed and wind power distributions in Rwanda

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ABSTRACT

Utilization of wind energy as an alternative energy source may offer many environmental and economical advantages compared to fossil fuels based energy sources polluting the lower layer atmosphere. Wind energy as other forms of alternative energy may offer the promise of meeting energy demand in the direct, grid connected modes as well as stand alone and remote applications. Wind speed is the most significant parameter of the wind energy. Hence, an accurate determination of probability distribution of wind speed values is very important in estimating wind speed energy potential over a region. In the present study, parameters of five probability density distribution functions such as Weibull, Rayleigh, lognormal, normal and gamma were calculated in the light of long term hourly observed data at four meteorological stations in Rwanda for the period of the year with fairly useful wind energy potential (monthly hourly mean wind speed $\bar{v} \geq 2 \quad \mathrm{m} \quad \mathrm{s}^{-1}$). In order to select good fitting probability density distribution functions, graphical comparisons to the empirical distributions were made. In addition, RMSE and MBE have been computed for each distribution and magnitudes of errors were compared. Residuals of theoretical distributions were visually analyzed graphically. Finally, a selection of three good fitting distributions to the empirical distribution of wind speed measured data was performed with the aid of a χ^2 goodness-of-fit test for each station.

Contents

1. Introduction

Wind energy, solar energy, geothermal energy and micro-hydro power energy are clean renewable solution energy sources that offer the promise of meeting energy demand in the direct, grid connected modes as well as stand alone and remote applications. Technologies utilized from wind resource for energy conversion

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Table 1
Geographical coordinates and elevations of the four observatories of the study.

Observatory	φ	λ	h (m)
GISENYI	01°40′S	29°15′E	1554 m
KIGALI	01°58′S	30°08′E	1490 m
BUTARE	02°36′S	29°44′E	1760 m
KAMEMBE	02°48′S	28°89′E	1487 m

φ: latitude, λ: longitude, h: elevation.

Table 2
Selected periods of the study with fairly useful wind energy potential for the four observatories of the study (monthly hourly mean wind speed $\bar{v} \geq 2\ \text{m s}^{-1}$).

Observatory	Period of observations	Selected period of the year	Selected period of the day
GISENYI	1981–1993	January–December	10:00–17:00 h
KIGALI	1981–1991	January–December	11:00–17:00 h
BUTARE	1988–1993	September–April	11:00–17:00 h
KAMEMBE	1881–1993	April–October	11:00–17:00 h

systems are well-established worldwide to provide complete security of energy supply [1–9].

Wind energy is the subsequent form of solar energy. It is a mechanical energy transported by an air current caused by the different distribution of solar heating of the Earth's surface by the Sun, thus generating the balance between pressure and temperature differences. Wind energy can be produced anywhere the wind blows with consistent force. A wind energy system transforms the kinetic energy of the wind into electrical or mechanical energy that can be exploited for practical use.

During the last two decades, in particular for wind energy, a number of studies have been conducted on electricity generation by means of wind turbine and on water pumping by direct mechanical means from wind energy conversion systems [12–20].

In Rwanda, quite few studies have been conducted on wind energy resource and yet wind energy potential in Rwanda has not been totally exploited for power generation though potential wind power that Rwanda possesses in some parts may offer possible solutions to electricity generation, water pumping and windmill [21,22]. Recently, the ministry of energy commissioned a feasibility study to determine the wind power capacity of Rwanda.

In practice, in order to optimize the design of wind energy conversion systems, it is imperative to determine wind energy potential for the selected site by investigating detailed knowledge of the wind characteristics, such as speed, direction, continuity, steadiness and seasonal variability. Thus, proper wind energy conversion systems resulting in less energy generating costs can be obtained. Wind speed distribution mainly provides the performance of wind power systems. Once the wind speed distribution is known, the wind power potential and, consequently, the economic feasibility could be easily obtained. While the hourly time-series wind speed data may always be huge, a small number of key parameters can allow determining wind characteristics and wind power potential of such a wide range of wind speed data by means of a distribution function. In literature, many studies base on their statistical analysis of wind characteristics and wind energy potential on the assumption that the Weibull distribution (or its special case, the Rayleigh distribution) approximates wind speed [22–36]. The reason is because of the easy estimation of its parameter to approximate the empirical distribution of wind observations. In a recent study [24], three other distributions were compared to the Weibull distribution. It was found, with a χ^2 goodness-of-fit test at the significance level of 0.05, that all three models give a good approximation irrespective of the orographic environment for k values significantly higher than 2. The Weibull distribution was recommended, due to the difficulty of parameter determination for the other models. In another study [25], ten probability distributions were compared using three goodness-of-fit tests at 95% confidence level and 5% significance. The Weibull distribution was identified to be the best distribution representing the wind data.

In this study, we tried to categorize appropriate theoretical probability density distributions of wind speed at four meteorological observatories in Rwanda. Five probability density distributions were tested and compared in order to determine the best fitting to empirical distribution of wind speed: the Weibull distribution, the Rayleigh distribution, the log-normal distribution, the normal distribution and the Gamma distribution.

2. Data and methods

2.1. Data

The National Meteorological Service is responsible for the Rwandan synoptic stations, and provides data records. Time-series of

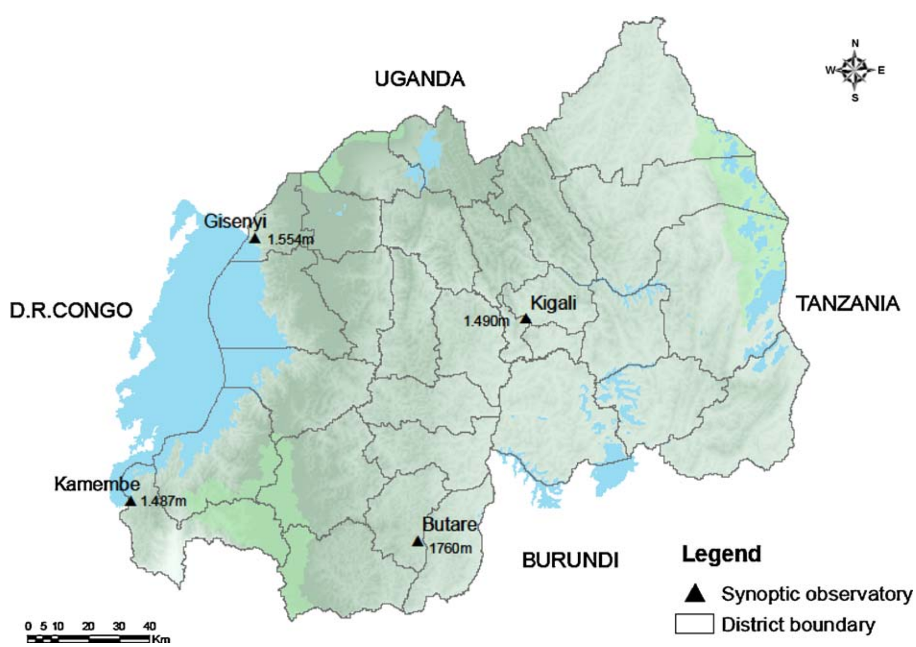


Fig. 1. Geographical locations of the four synoptic observatories.

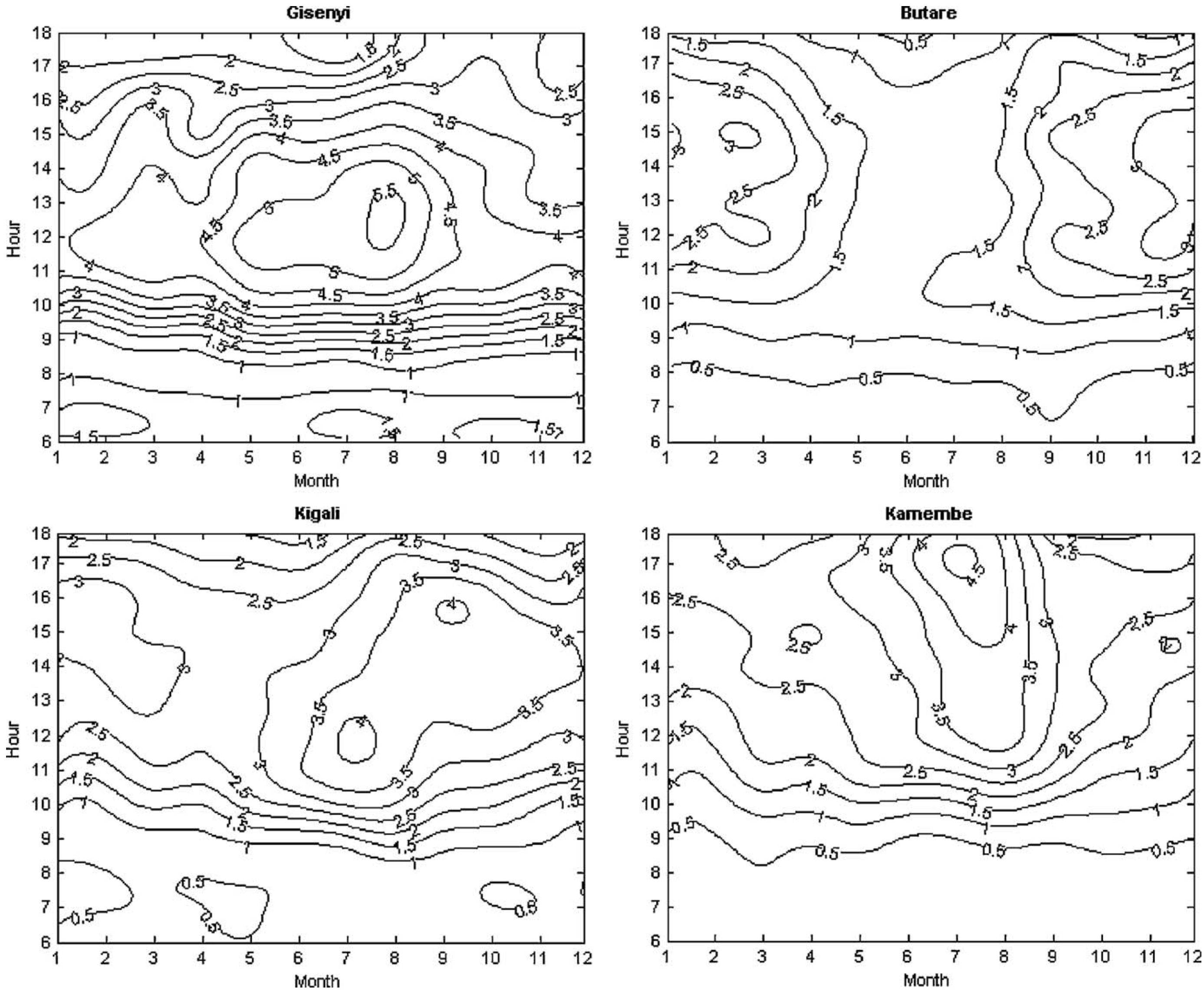


Fig. 2. Monthly hourly mean wind speed in m s^{-1} at the four observatories of the study.

measured hourly daily wind speed and wind direction data were supplied by the National Meteorological Service. The data for the four stations, Gisenyi, Kigali, Butare and Kanombe were chosen for this study because they had the largest long term average wind speeds among the six meteorological stations in Rwanda. Fig. 1 and Table 1 present the geographical coordinates and elevations of the four stations of the study. All of these stations are located in the local airports with windmill type anemometers installed at 10 m above ground level. Data were selected in the long term time series of hourly wind speed and wind direction data for the period of the year with fairly useful wind energy potential (monthly hourly mean wind speed $\bar{v} \geq 2\ \ \text{m}\ \ \text{s}^{-1}$) as indicated in Table 2.

2.2. Methods

2.2.1. Data processing

A two-dimensional cubic spline interpolation using Matlab 6.5.1 was performed and applied to the computed monthly hourly mean wind speed obtained from the long term hourly wind speed measured data, and the results were then plotted in an orthogonal three dimensional graphics as presented in Fig. 2. Sixteen directional wind rose presenting frequencies of direction of each wind speed for each station and for the period indicated in Table 2 was processed and plotted with the use of WRPLOT View 5.9® 1998–2008 Lakes Environmental Software as shown in Fig. 3.

2.2.2. Statistical probability density functions

The probability density distribution functions below were tested to approximate frequency distribution of hourly wind speed data for the considered period of each observatory.

2.2.2.1. The Weibull probability density function. A random variable v, here the wind speed, has a Weibull distribution if its probability density function is defined by [22]:

$$\begin{cases} f(v) = f(v;\quad k, c) = \dfrac{k}{c}\left(\dfrac{v}{c}\right)^{k-1} e^{-(v/c)^k}, \quad v > 0, k, c > 0 \\ f(v) = 0, \quad v \leq 0 \end{cases} \tag{1}$$

where k is a so-called shape parameter (a dimensionless number) and c is scale parameter (m s^{-1}).

If $k = 2$, then we have a special case of the Weibull distribution called the Rayleigh distribution [10,11] whose distribution density

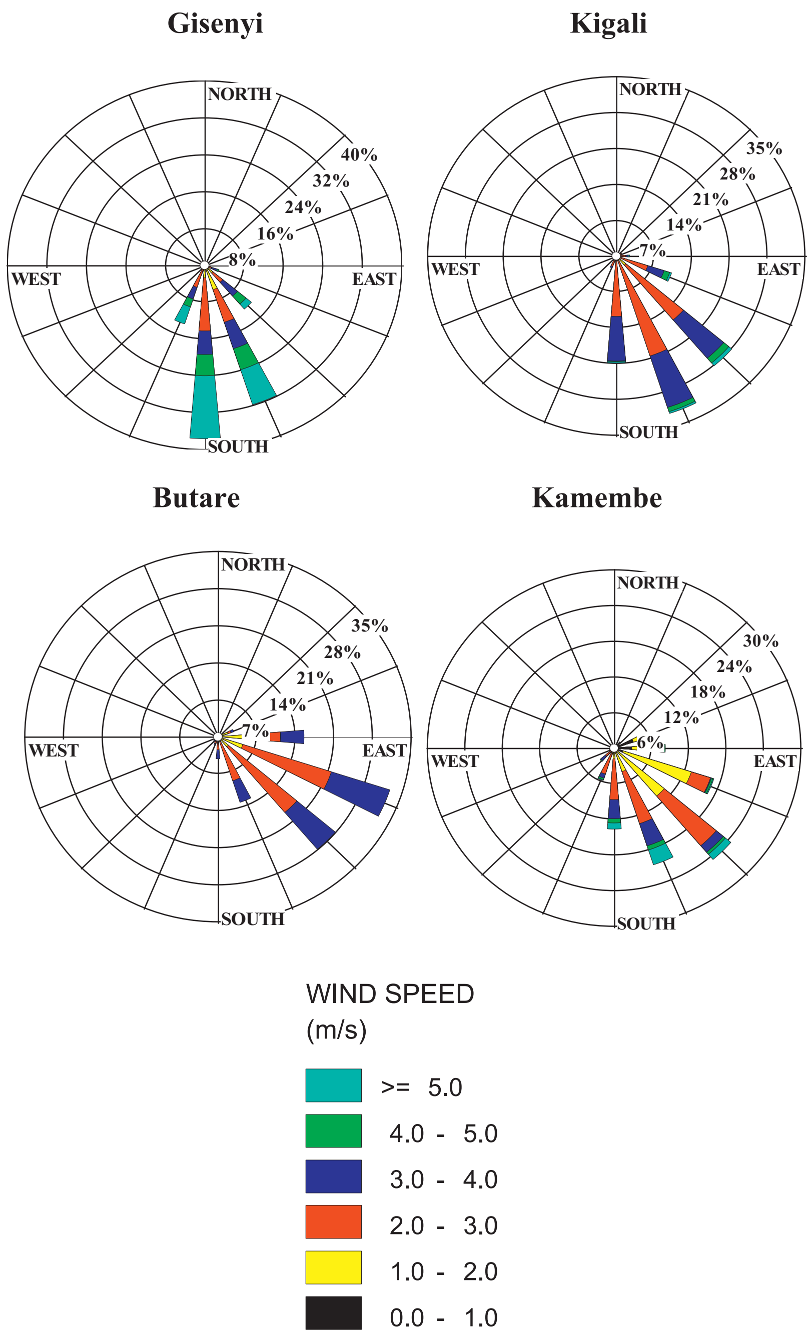


Fig. 3. Wind roses at the four observatories of the study for the respective periods of study with fairly useful wind (monthly hourly mean wind speed $\bar{v} \geq 2\ \text{m s}^{-1}$).

is:

$$f(v) = f(v;\quad c) = \frac{2v}{c^2} e^{-(v/c)^2} \tag{2}$$

With such a distribution, the expected value of a probability variable is:

$$\mu = \frac{c\sqrt{\pi}}{2} \tag{3}$$

In this situation, the scale parameter c is in proportion to the average.

The Weibull cumulative distribution function is given by:

$$F(v;\quad k, c) = 1 - e^{-(v/c)^k} \tag{4}$$

2.2.2.1.1. Estimation of the Weibull parameters k and c. Various methods have been developed for estimating the parameters of the Weibull probability distribution function. The most commonly used have been the method of moments [24,27,28], the maximum likelihood method [26], the least square method [24,28,29] and Chi-square method [28]. However, the maximum likelihood method has proved to be the most efficient [26,27] in determining the parameters of Weibull probability distribution function. In order to determine the best method to be used in the present study, the two mostly used methods in estimation of parameters, the least square and maximum likelihood have been compared.

2.2.2.1.2. The least square or regression method. The cumulative distribution function can be linearized by taking the natural logarithm of both sides of Eq. (4) twice as follows:

$$\ln[-\ln\{1 - F(v)\}] = k\quad \ln(v) - k\quad \ln(c) \tag{5}$$

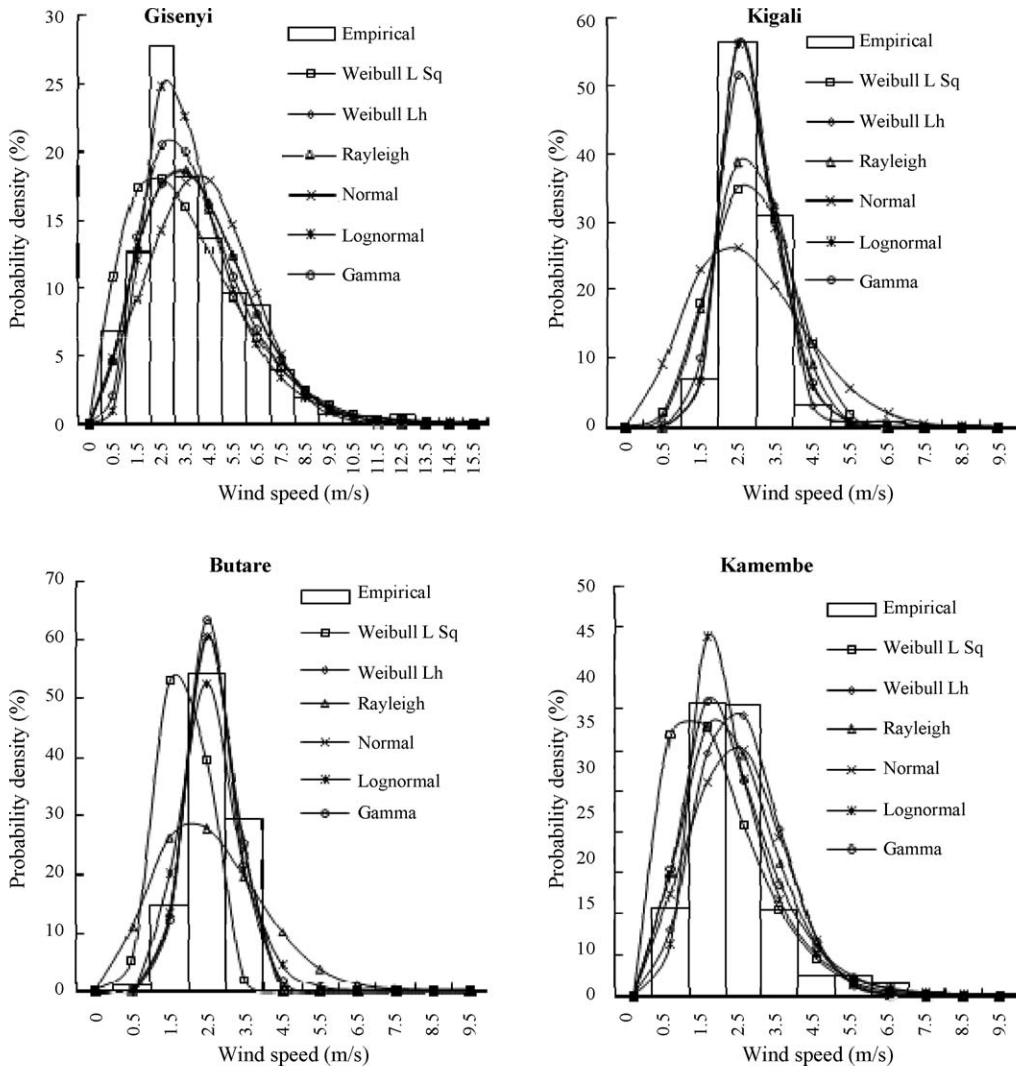


Fig. 4. Fittings of the wind speed probability density distributions with the empirical distributions for the respective periods of study at the four observatories.

The parameters in the resulting equation are obtained using the least squares method. A plot of $\ln[-\ln[1-F(v)]]$ versus $\ln(v)$ presents a straight line. If a and b are the estimators of the intercept and slope from the regression equation, then the scale and shape parameter estimators are $\hat{k}=b$ and $\hat{c}=\exp(-a/\hat{k})$.

2.2.2.1.3. Maximum likelihood method. Assume $(v_1, v_2, \ldots, v_n)$ is a random sample with a probability density function, here the Weibull function, of the form given by Eq. (1). The likelihood function of the random sample $(v_1, v_2, \ldots, v_n)$ denoted by $L(k, c, v_1, v_2, \ldots, v_n)$ is the joint density of the variables involved, that is:

$$L(k, c, v_1, v_2, \ldots, v_n) = \prod_{i=1}^{n} f(k, c, v_i) \quad (6)$$

Then, we have:

$$\ln \ L = \sum_{i=1}^{n} \ln[f(v_i)] = n[\ln \ k - k \ \ln \ c] + (k-1)\sum_{i=1}^{n} \ln(v_i) - c^{-k}\sum_{i=1}^{n} (v_i)^k \quad (7)$$

For n independent data $v_1, v_2, \ldots, v_n$ of variable v, the maximum of the function Eq. (7) is determined by solving the following system of equation:

$$\begin{cases} \dfrac{\partial \ \ln \ L}{\partial k} = 0 \\ \dfrac{\partial \ \ln \ L}{\partial c} = 0 \end{cases} \quad (8)$$

Solutions of Eq. (8) must satisfy the following system of equations:

$$\begin{cases} \hat{c} = \left(\dfrac{1}{n}\displaystyle\sum_{i=1}^{n} v_i^{\hat{k}}\right)^{1/\hat{k}} \quad (a) \\ \dfrac{n}{\hat{k}} - n \ \ln(\hat{c}) + \displaystyle\sum_{i=1}^{n} \ln(v_i) - \sum_{i=1}^{n}\left(\dfrac{v_i}{\hat{c}}\right)^{\hat{k}} \ \ln\left(\dfrac{v_i}{\hat{c}}\right) = 0 \quad (b) \end{cases} \quad (9)$$

By eliminating $\hat{c}$ from the system of Eq. (9), we obtain the following equation which gives the value of $\hat{k}$ from which the value of $\hat{c}$ can be obtained by Eq. (9(a)):

$$\hat{k} = \left[\frac{\sum_{i=1}^{n} v_i^{\hat{k}} \ln(v_i)}{\sum_{i=1}^{n} v_i^{\hat{k}}} - \frac{1}{n}\sum_{i=1}^{n} \ln(v_i)\right]^{-1} \quad (10)$$

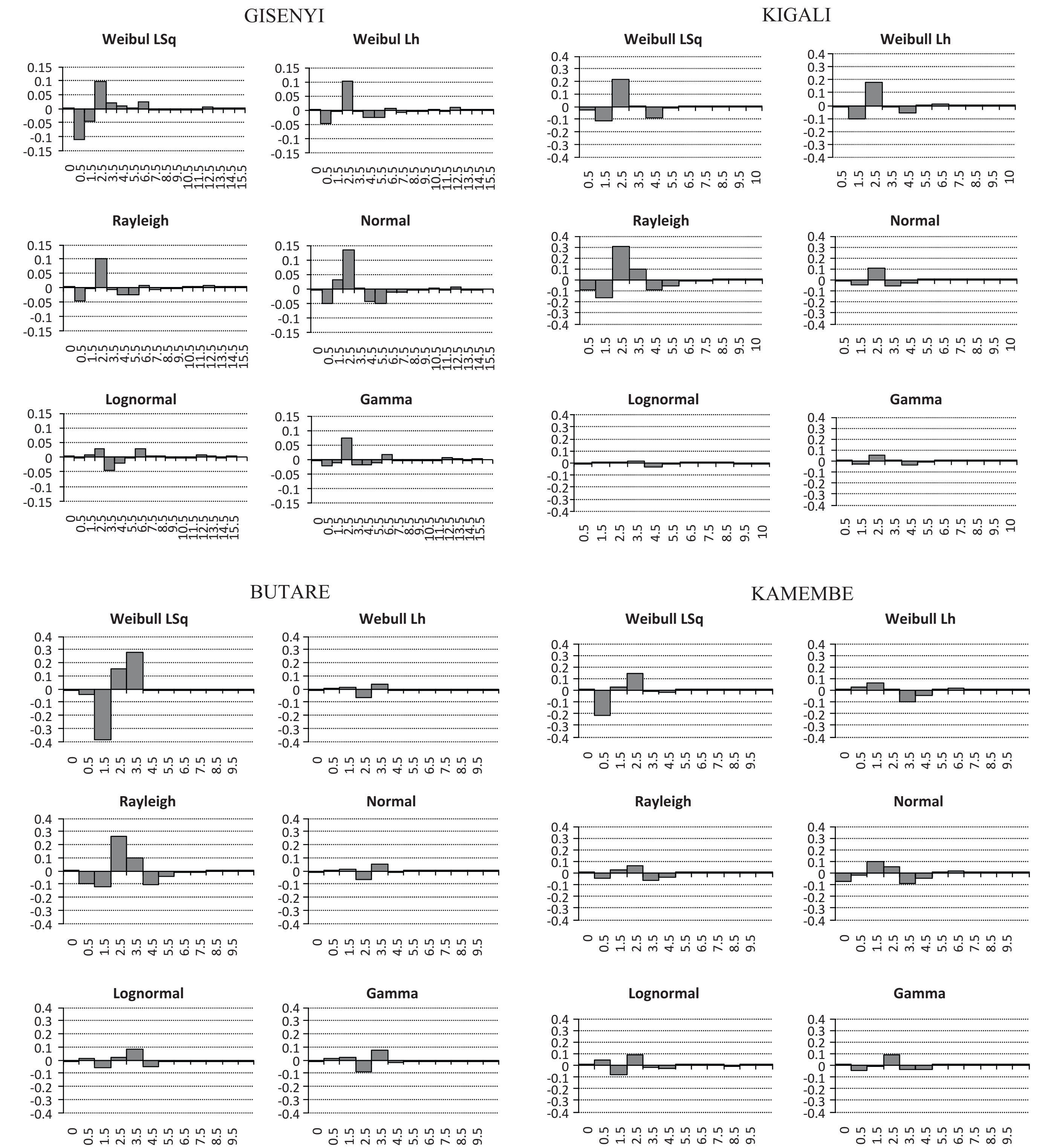


Fig. 5. Residuals of the wind speed probability density distributions at the four observatories for the respective period of study.

Eq. (10) is solved with an iterative method starting with the value $\hat{k}_0$ given by [25]:

$$\hat{k}_0 = \left(\bar{v}\sqrt{var}\right)^{-1.086} \tag{11}$$

where $\bar{v}$ and $\sqrt{var}$ are respectively the sample mean and variance of the series.

2.2.2.2. Normal distribution.

$$f(v) = f(v;\quad \mu, \sigma) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(v-\mu)^2/2\sigma^2} \tag{12}$$

Its parameters are the μ expected value and the σ standard deviation of the v probability variable.

Table 3
Parameters of wind speed probability density distributions at the four observatories for the respective period of study.

Observatory	Weibull likelihood		Weibull least square		Rayleigh		Normal		Lognormal		Gamma	
	k	c	k	c	k	c	μ	σ	μ	σ	p	λ
GISENYI	1.99	4.58	1.57	4.14	2	4.57	4.04	2.18	1.27	0.50	3.44	0.85
KIGALI	3.31	3.15	2.99	3.23	2	3.23	2.86	0.78	1.02	0.26	13.29	4.64
BUTARE	4.93	2.86	3.37	2.09	2	2.96	2.62	0.64	0.93	0.30	0.93	0.30
KAMEMBE	2.38	2.74	1.48	2.07	2	2.55	2.25	1.30	0.66	0.54	2.99	1.33

Table 4
Errors (RMSE and MBE (%)) of wind speed probability density distributions to the empirical wind speed distributions at the four observatories for the respective period of study.

Observatory	Weibull likelihood		Weibull least square		Rayleigh		Normal		Lognormal		Gamma	
	RMSE	MBE	RMSE	MBE	RMSE	MBE	RMSE	MBE	RMSE	MBE	RMSE	MBE
GISENYI	0.028	−0.23	0.038	−0.57	0.029	−0.25	0.040	−0.47	0.016	−0.0043	0.021	−0.14
KIGALI	0.072	0.023	0.087	0.027	0.13	0.11	0.044	0.026	0.012	−0.054	0.023	0.0053
BUTARE	0.034	−0.16	0.22	−0.036	0.14	−2.9	0.037	−0.17	0.047	−1.25	0.045	−0.53
KAMEMBE	0.039	0.041	0.080	0.28	0.062	0.12	0.050	0.28	0.040	−0.27	0.034	0.090

Table 5
Good fittings at the significance level of 0.05 (+) for the periods of study.

Observatory	Weibull		Rayleigh	Normal	Lognormal	Gamma
	Likelihood	Least Square				
GISENYI	+		+		+	+
KIGALI				+	+	+
BUTARE	+			+		+
KAMEMBE	+		+		+	+

Table 6
Best fitting probability density functions for the four sites.

Observatory	First	Second	Third
GISENYI	Gamma (p = 3.44; λ = 0.85)	Lognormal (μ = 1.27; σ = 0.50)	Weibull (k = 1.99; c = 4.58)
KIGALI	Gamma (p = 13.29; λ = 4.64)	Normal (μ = 2.86; σ = 0.78)	Lognormal(μ = 1.02; σ = 0.26)
BUTARE	Weibull (k = 4.93; c = 2.86)	Normal (μ = 2.62; σ = 0.64)	Gamma (p = 16.60; λ = 6.33)
KAMEMBE	Gamma (p = 2.99; λ = 1.33)	Lognormal (μ = 0.66; σ = 0.54)	Weibull (k = 2.38; c = 2.74)

2.2.2.3. *Log-normal distribution.*

$$f(v) = f(v;\quad \mu, \sigma) = \frac{1}{\sigma v\sqrt{2\pi}} e^{-(\ln\quad v-\mu)^2/2\sigma^2} \tag{13}$$

Its parameters are the μ expected value and the σ standard deviation of the ln v probability variable ($v > 0$).

2.2.2.4. *Gamma distribution.*

$$\begin{cases} f(v) = f(v;\quad \lambda, p) = \dfrac{\lambda^p}{\Gamma(p)} v^{p-1} e^{-\lambda v}, & v > 0 \\ f(v;\quad \lambda, p) = 0, & v \le 0 \end{cases} \tag{14}$$

where $\Gamma(p)$ is the gamma function. The μ expected value and the σ^2 of the standard deviation of a probability variable with such a distribution is:

$$\mu = \frac{p}{\lambda}, \qquad \sigma^2 = \frac{p}{\lambda^2}$$

2.2.3. *Wind speed variation with height above the Earth's surface*

In general, wind speed measurements are made at a standard altitude such as 10 m above the Earth's surface. For projects involving wind conversion system, it is required to estimate wind speeds at various elevation. The wind speed increases with height. When record of wind speed exists at different height for a station, the commonly power law can be used to obtain the extrapolated values of wind speed at different heights [23,31]:

$$v = v_a\left(\frac{z}{z_a}\right)^\alpha \tag{15}$$

Table 7
Comparison of the computed empirical power density with the power densities obtained from the three selected probability density functions.

Observatory	Empirical	Weibull	Normal	Lognormal	Gamma
GISENYI	85.8	80.1 k = 1.99; c = 4.58		79.4 μ = 1.27; σ = 0.50	81.5 p = 3.44; λ = 0.85
KIGALI	18.9		17.6 μ = 2.86; σ = 0.78	17.8 μ = 1.02; σ = 0.26	17.8 p = 13.29; λ = 4.64
BUTARE	13.3	12.7 k = 4.93; c = 2.86	12.5 μ = 2.62; σ = 0.64		12.0 p = 16.60; λ = 6.33
KAMEMBE	17.1	14.4 k = 2.38; c = 2.74		15.5 μ = 0.66; σ = 0.54	15.3 p = 2.99; λ = 1.33

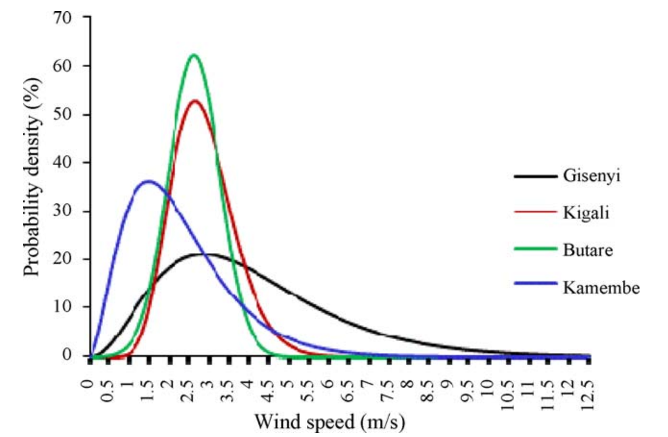


Fig. 6. Best wind speed probability density distribution functions for the four observatories gamma (p = 3.44; λ = 0.85) for Gisenyi, gamma (p = 13.29; λ = 4.64) for Kigali, Weibull (k = 4.93; c = 2.86) for Butare and gamma (p = 2.99; λ = 1.33) for Kamembe.

where v_a is the wind speed measured at anemometer height z_a, v is the wind speed to be calculated at the height z, α is the power law exponent depending on the surface roughness and obtained empirically. The value of the coefficient α varies from less than 0.10 at the summits of steep hills to more than 0.25 in sheltered locations [31–33].

Another technique uses the Weibull probability density function to obtain the extrapolated values of wind speed at different heights. Parameters of Weibull distribution functions k_z and c_z for altitudes z above the anemometer level are obtained using the following relations [24,29]:

$$k_z = \frac{k_a[1 - 0.088 \quad \ln(z_a/10)]}{[1 - 0.088 \quad \ln(z/10)]} \tag{16}$$

$$c_z = c_a\left(\frac{z}{z_a}\right)^n \tag{17}$$

where k_a and c_a are respectively the shape parameter and the scale parameter at the anemometer height z_a and the exponent n is given by the relation:

$$n = \frac{[0.37 - 0.088 \quad \ln \quad c_a]}{[1 - 0.088(\ln \quad z_a/10)]} \tag{18}$$

2.2.4. Evaluation of power density distribution

The most important wind characteristic is the wind energy density. Assume A is a cross-section through which the wind of speed v flows perpendicularly. The available wind power is defined as the flow of kinetic energy which is obtained by the relation [24–26]:

$$P(v) \quad (\mathrm{W}) = (\tfrac{1}{2}v^2)vA = \tfrac{1}{2}\rho v^3 A \tag{19}$$

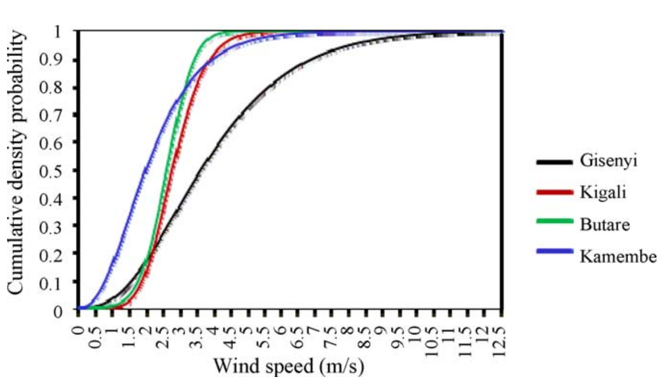


Fig. 7. Cumulative wind speed probability density distribution functions for the four observatories.

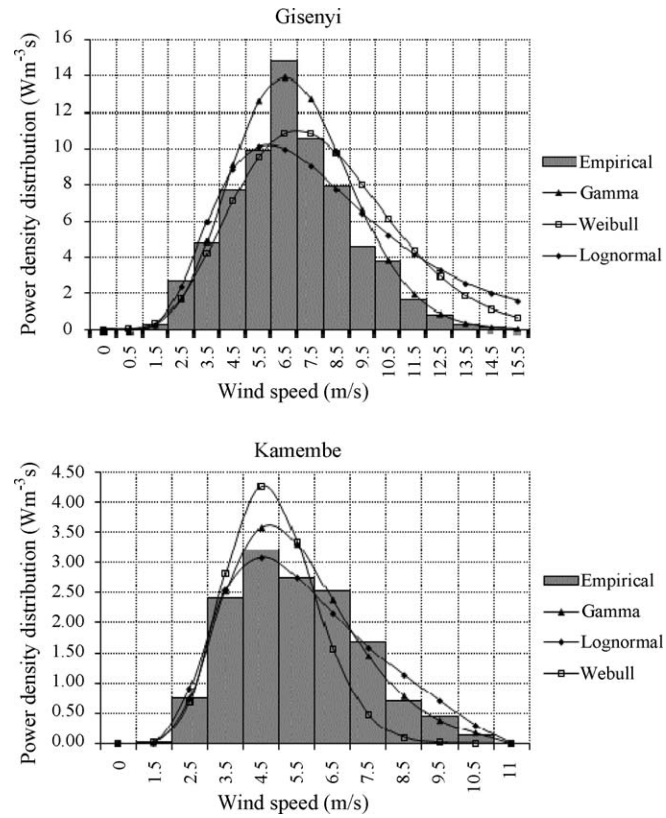


Fig. 8. Comparison of the three wind power density distributions obtained using the three selected wind speed probability density distributions for Gisenyi and Kamembe.

where ρ is the air density which depends on pressure (altitude), temperature and humidity. It is assumed to be constant since its variation does not affect significantly wind resource calculation [24–26]. The commonly used value is $\bar{\rho} = 1.225 \quad \mathrm{kg} \quad \mathrm{m}^{-3}$ corresponding to standard conditions (sea level, 15 °C).

The power density distribution gives the distribution of wind energy at different wind speeds. It is obtained by multiplying the wind power density with the value corresponding to the probability density function $f(v)$ of each wind speed v as follows:

$$\frac{P(v)}{A}f(v) \quad (\mathrm{W} \quad \mathrm{m}^{-3} \quad \mathrm{s}) = \tfrac{1}{2}\bar{\rho}v^3 f(v) \tag{20}$$

By integrating Eq. (19) for the period of study we obtain the mean wind power density:

$$\bar{P} \quad (\mathrm{Wm}^{-2}) = \frac{1}{2}\bar{\rho}\int_0^\infty v^3 f(v)dv \tag{21}$$

3. Results and discussion

3.1. Monthly hourly mean wind speed and wind direction analysis

In the present study, time-series of hourly measured wind speed and wind direction data of four sites have been statistically analyzed. Monthly hourly mean wind speed values calculated from the available data are presented in Fig. 2. Daily wind density variations are directly related to daily temperature variations. The variations are low in the mornings, reach a maximum value in the afternoons, and start decreasing in the evenings. As a result, higher wind speeds occur between 10.00 and 11.00 a.m. and 15.00 and 17.00 p.m. due to the increasing of the thermal gradient generated by the solar heating of the Earth's surface, reaching its maximum value around

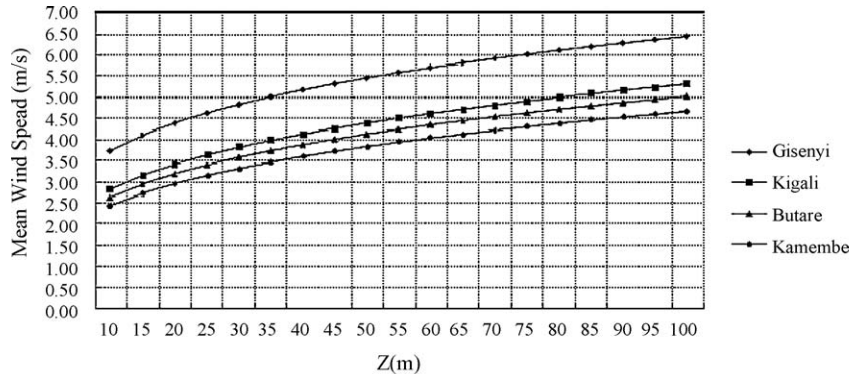


Fig. 9. Computed mean wind speeds at various height above the Earth's surface (anemometer level 10 m) for the period of study of each observatory.

13.00–14.00 p.m. The seasonal variability of the wind speed is well marked showing for the sites of Gisenyi and Kamembe, higher wind speed values during the period between June and September, corresponding to the dry season. The two sites are located on the eastern coast of the Lake Kivu and consequently the cooling effect of the continent causes higher wind speed during that period of the year. The sites of Kigali and Butare are continental and the wind flow, trade winds, which is relatively low due the latitudinal locations of the two sites closed to the Equator, follows the seasonal cycle of the Intertropical Convergence Zone (ITCZ) with maximum values during the rainy seasons (October–December and January–April). The charts of the wind roses observed in Fig. 3 demonstrate that most of the wind flows from South to Southeast: (-77.5° to 90°) for Gisenyi; (-45° to 90°) for Kigali; (22.5° to 45°) for Butare and (22.5° to 77.5°) for Kamembe.

3.2. *Wind speed probability density distribution functions*

Parameters of estimated statistical distributions and errors were computed and are presented in Table 3 for the four stations. The RMSE and MBE (%) are, respectively, the root mean square and the mean biased error computed from wind speed probability density functions relatively to the empirical frequency distribution for the respective period of study.

Graphical comparisons of those statistical distributions to the empirical distributions were made as illustrated in Fig. 4. Residuals of theoretical distributions have been computed and areas on the graphics presented in Fig. 5 show differences. Large areas on the graphics show that the residual of the measured and estimated values is great. Therefore, a decrease of the residuals corresponds to an increase of ability for the statistical distribution to best fit the empirical distribution of the data sets.

For the station of Gisenyi, the lognormal ($\mu = 1.27$; $\sigma = 0.50$) distribution corresponds to the best fit of data set followed by the gamma ($p = 3.44$; $\lambda = 0.85$) distribution and the Weibull ($k = 1.99$; $c = 4.58$) distribution. For the station of Kigali, the lognormal ($\mu = 1.02$; $\sigma = 0.26$) corresponds to the best fit of data set followed by the gamma ($p = 13.29$; $\lambda = 4.64$) distribution and normal ($\mu = 2.86$; $\sigma = 0.78$). For the station of Butare, the Weibull ($k = 4.93$; $c = 2.86$) distribution corresponds to the best fit of data set followed by the Normal ($\mu = 2.62$; $\sigma = 0.64$) and the gamma ($p = 16.60$; $\lambda = 6.33$). For the station of Kamembe, gamma ($p = 2.99$; $\lambda = 1.33$) distribution corresponds to the best fit of data set followed by the Weibull ($k = 2.38$; $c = 2.74$) and the lognormal ($\mu = 0.66$; $\sigma = 0.54$).

The selected probability distribution functions were compared for decision, with the aid of a χ^2 goodness-of-fit test at 95% confidence level and 5% significance level, to the empirical distribution of wind speed measured data. Good fittings at the significance level of 0.05 for the periods of study are presented in Table 4. The + sign indicates cases where the approximation proved good at least the abovementioned significance level. The order of best fitting distributions was reorganized giving priority to the highest value of the level of significance for acceptance (≥ 0.05) for each approximation. The results are presented in presented in Table 5.

3.3. *Spatial variability of wind speed probability density distribution functions*

From the above analysis, it is understandable that the wind speed probability density distributions are quite different for different sites, so it is very imperative to select a suitable site with good wind field for wind power generation. The best selected fitting wind speed probability distributions are presented in Fig. 6 for the four sites and for respective periods of the year considered in

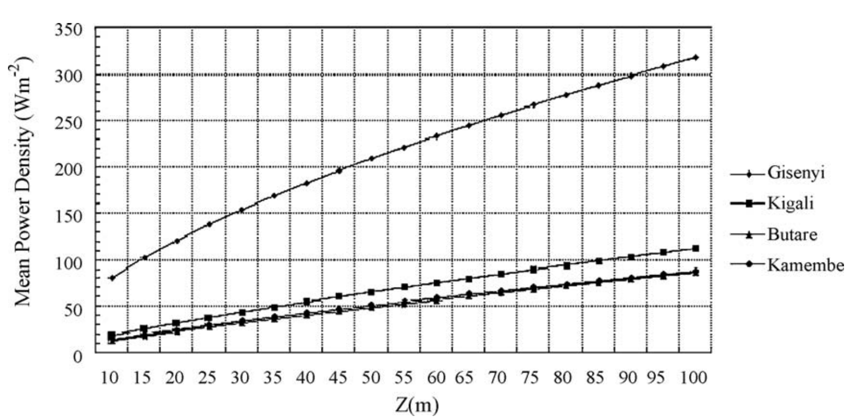


Fig. 10. Computed mean power density at various height above the Earth's surface (anemometer level 10 m) for the period of study of each observatory.

the present study. The results show that Gisenyi is the windiest site having more opportunities of experiencing wind speeds above 4.5 m s^{-1} than other sites. Butare has the most peaked wind distribution at 2.62 m s^{-1} with a probability of 61.9% and an average wind speed of 2.62 m s^{-1}. The highest-probability wind speed of Kigali is 2.65 m s^{-1} at 52.5%. Butare has comparable wind speed conditions to Kigali except that it has slightly more frequency of lower wind speeds and less frequency of higher wind speeds. Kamembe appears to have the lowest wind value 1.3 m s^{-1} at its highest probability distribution 35.3%, but it experiences a little more frequency of higher wind speeds than Butare.

Cumulative distribution functions are presented in Fig. 7. If 3 m s^{-1} and 25 m s^{-1} are used as the cut-in and cut-out speeds for wind turbines, Gisenyi will have the highest operating possibility of 74% (2920 h/year); Kigali, Butare and Kamembe will have 65% (2555 h/year), 58% (1694 h/year) and 35% (1477 h/year) operating possibilities, respectively.

3.4. Wind power density distribution and mean power density

The wind energy density is a function of the distribution of wind speeds and the effect of air density. The power density distribution generated by the three selected wind speed probability density functions is presented in Fig. 8. The three selected wind power density distributions compared to the empirical power frequency distribution reveal similar characteristics at the lower wind speeds. Whereas at the higher wind speeds, the wind power density distributions have different characteristics. This is due to the fact that the wind power is proportional to the cube of wind speed and therefore, minor errors in the estimation of the wind speed probability density distribution function in higher wind speed range can cause larger discrepancies in the wind power density distribution function.

Comparisons of the wind power density distribution to the empirical power frequency distribution and the mean power density to the empirical mean power density confirm the order of selection of the wind speed probability density functions.

The computed values of power density from the selected distributions (Table 6) for the four sites are presented in Table 7. The results reveal that the station of Gisenyi possesses suitable wind energy potential for electricity generation whereas the stations of Kigali, Butare and Kamembe have sufficient energy potential for windmills or water pumping.

3.5. Wind speed and power density variation with height above the Earth's surface

In Figs. 9 and 10, we present the results obtained from computed mean wind speeds and mean power density at various height above the Earth's surface (anemometer level 10 m) for the period of study of each observatory. Gisenyi is found to have very good values of mean wind speed and mean power density at high altitudes. Whereas Kigali and Butare are found to have relatively good values of mean wind speed and mean power density at high altitudes. The available wind energy can therefore be harnessed to generate electricity and mechanical energy for windmills or water pumping in Kigali, Butare and Gisenyi.

4. Conclusion

Rwanda possesses potential wind power in some parts which may offer possible solutions to electricity generation, water pumping and windmill. In this study, we tried to categorize appropriate theoretical probability density distributions of wind speed at four meteorological observatories in Rwanda. To evaluate the performance of the considered distributions, root-mean-square error (RMSE) and Mean biased error (MBE) parameter were used in the study. Graphical comparisons of the distributions and residuals have proven they are supporting the above methods. With the means of χ^2 goodness-of-fit test for each station, a selection of three good fitting distributions to the empirical distribution of wind speed measured data was performed with the aid of a χ^2 goodness-of-fit test for each station. According the test carried out, the following distribution functions obtained for the investigated four sites are the more suitable: gamma (p = 3.44; λ = 0.85) followed by lognormal (μ = 1.27; σ = 0.50) and Weibull (k = 1.99; c = 4.58) for the site of Gisenyi; Gamma (p = 13.29; λ = 4.64) followed by normal (μ = 2.86; σ = 0.78) and lognormal (μ = 1.02; σ = 0.26) for the station of Kigali; Weibull (k = 4.93; c = 2.86) followed by normal (μ = 2.62; σ = 0.64) and gamma (p = 16.60; λ = 6.33) for the site of Butare; gamma (p = 2.99; λ = 1.33) followed by lognormal (μ = 0.66; σ = 0.54) and Weibull (k = 2.38; c = 2.74) for the site of Kamembe. Power densities computed from the selected distributions for the four sites exhibit potential wind energy for electricity generation, windmills and water pumping for rural areas.

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